Matching Matrix Elements and Parton Showers with HERWIG and PYTHIA

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Work with P. Richardson (HERWIG)

Tools for Background Estimation

Parton Shower Event Generators

- soft-collinear emissions (all orders)
- direct connection to hadronization model
- trust shapes, not rate (structure of jets)

Matrix Element Samples of Unweighted Events

- hard and wide-angle emissions (fixed order)
- full phase space, interference
- partons have virtualities on the order of Q_F

Higher order calculations

- reduced sensitivity to factorization scale
- negative weights
- more inclusive predictions

Merging ME and PS: I

We want to use both in a consistent way

- ME gives hard/wide angle emissions
- PS gives soft/collinear emission
- Want smooth matching between the two
  - limit sensitivity to where matching occurs
- No double counting of emissions
- No under counting of emissions

X Exact NLO corrections are another story

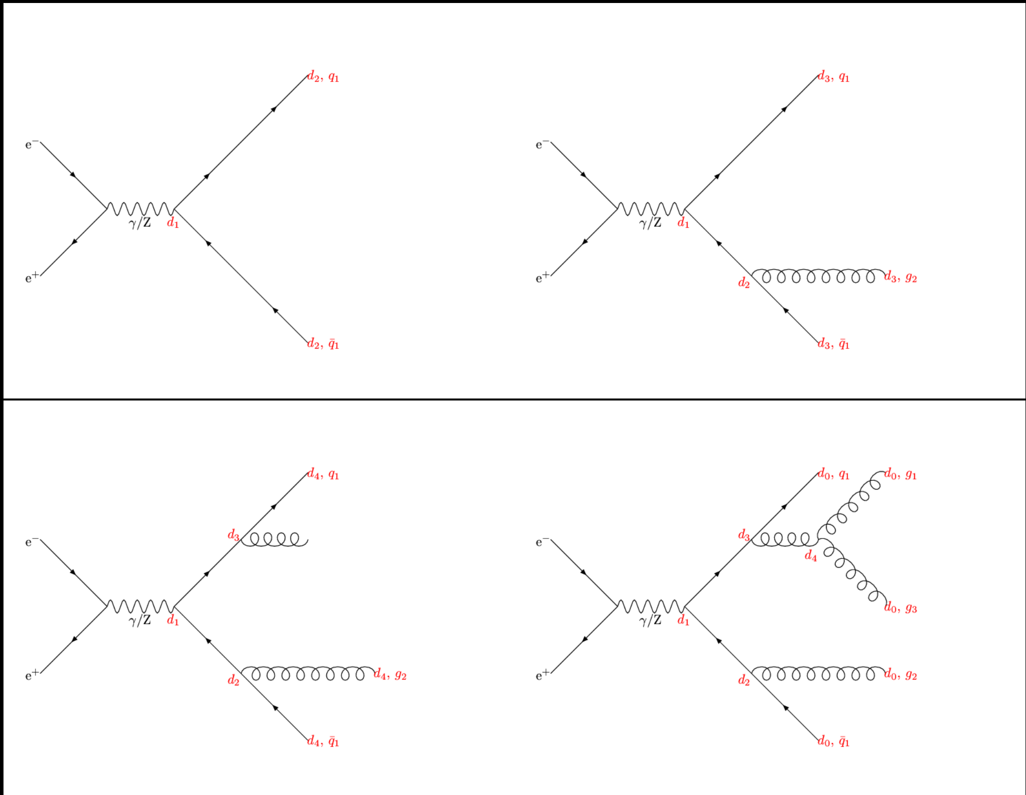
Merging ME and PS: II

- There have been a number of attempts to do this
- Hard emission corrections for relatively simple cases
  - $e^+e^- \longrightarrow q\bar{q}$
  - DIS
  - $\gamma^*/W/Z \longrightarrow$ leptons
  - Top Decay
  - PYTHIA (Sjöstrand, et al)+HERWIG (Seymour, et al)
  - Basic Idea:
    1. Rewrite (simple) ME^2 in terms of shower variables
    2. Reweight first emission to get this expression
- Only hardest (or first) emission correctly described
- Leading order normalization retained

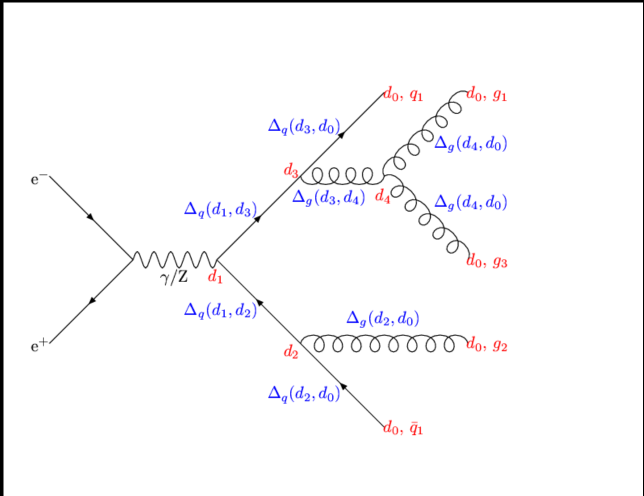
Recent Developments

- NLO Simulation (Frixione/Webber)
  - NLO normalization of the cross section
  - Shower unchanged, but gives the correction expansion to NLO
  - Passes negative weights (but total rate is positive)
  - Not generalizable
- Multijet Leading Order (Catani/Kuhn/Krauss/Webber; Lönnblad)
  - LO + NLL
  - Generalizes to many hard emissions
- Rest of talk on 2^{nd} approach

Anatomy of a Final State



Weight of the Final State

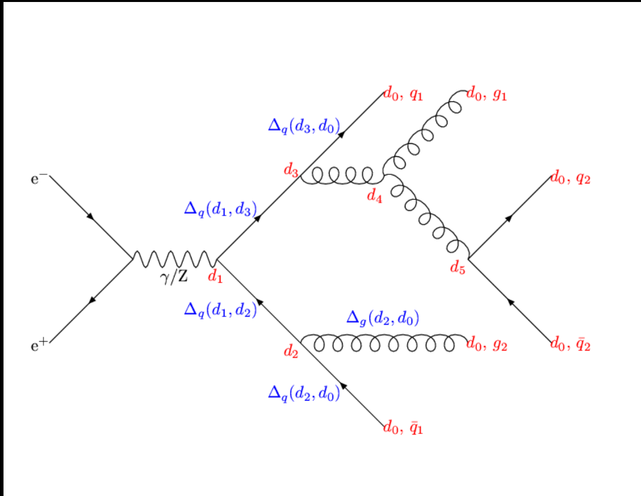


- Nodal values d_i represent decreases in virtuality
- Sudakov form factors $\Delta_{q,g}$ are probabilities for no emission
- $\alpha_s(d_i)P(z)$ at each splitting
- Shower is stopped at scale $d_0 \sim \Lambda_{QCD} \Rightarrow$ hadronization

The Correction Procedure of CKKW

0. Calculate (at ME level) tree level cross sections σ_n^0 for $e^+e^- \to q\bar{q}n$ and $n = 0, N$ at a resolution scale d_0
1. Select the jet multiplicity $P_n^{(0)} = \frac{\sigma_n^0}{\sum_{k=0}^{k=N} \sigma_k^0}$
2. Select particle momenta according to the $|\mathcal{M}_n|^2$
3. Cluster partons using the k_T-algorithm to give resolution values $d_1 > d_2 \ldots > d_n > d_0$
4. Reweight by $\alpha_S(d_1)\alpha_S(d_2)\cdots\alpha_S(d_n)/\alpha_S(d_0)^n$
5. Apply a NLL Sudakov weight factor $\Delta(d_k, d_j)$ on each line
6. Apply Monte Carlo rejection
7. PS accepted configuration vetoing all radiation with $d > d_0$. Starting scale of each PS is the scale at which the particle was created.

Add on a Vetoed Shower



- $\bar{q}_1$ starts shower at scale d_1
  - veto emissions with $d > d_0$
- Failure to do this causes a logarithmic dependence on cutoff scale relative to hadronization scale
  - destroys exponentiation

Practical Application

- Want to do this with PYTHIA and HERWIG
  - tested, trusted, integrated
- PYTHIA and HERWIG are not k_T-ordered showers
- Sudakovs are different/numerical/conserve energy-momentum
- Kinematics within shower not the same as at the end
- Ordering in virtuality sometimes in conflict with k_T

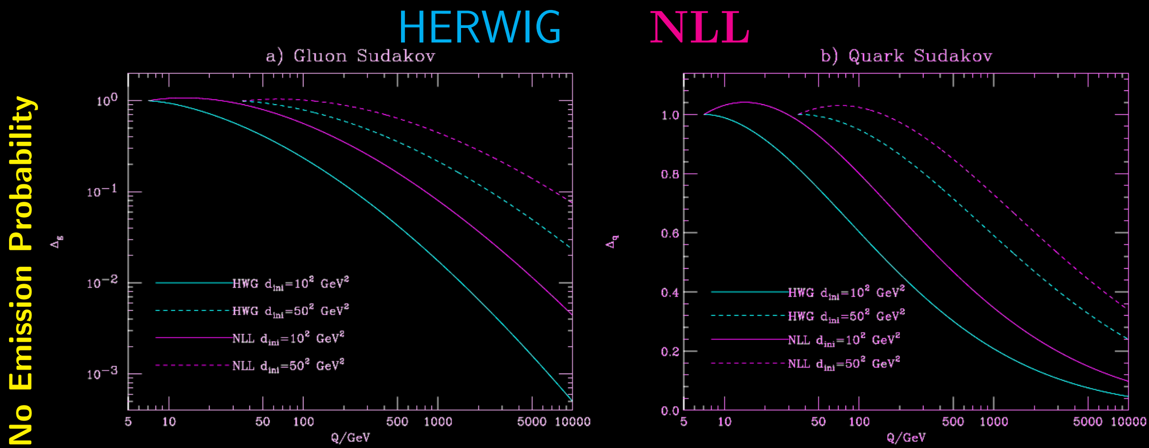
PR has continued development along CKKW lines

- Tries several scales, prefactors, minimum values to achieve stable results

I have developed an approach tailored to each generator

- Less freedom (choices made by generator)

Sudakovs

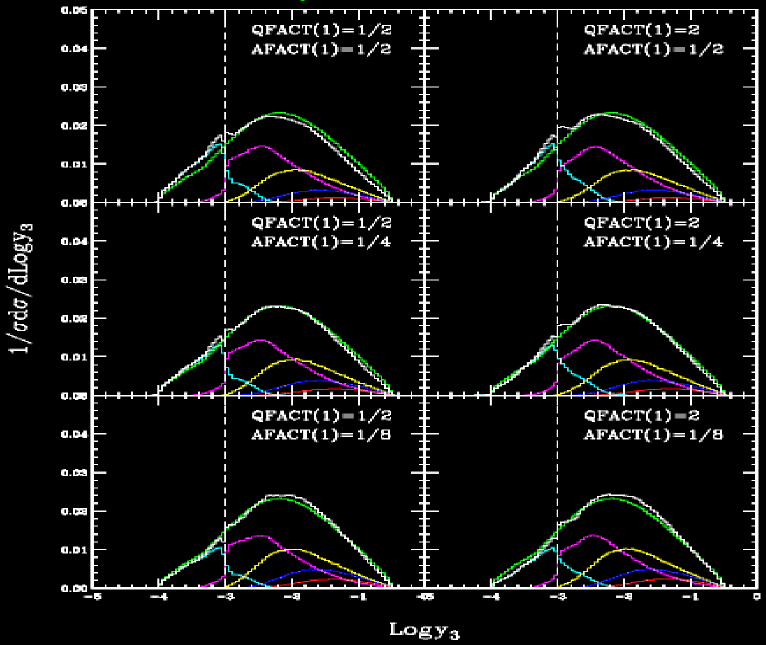


Solid: $d_0 = 10^2\ \mathrm{GeV}^2$ Dashed: $d_0 = 50^2\ \mathrm{GeV}^2$

- HERWIG has energy-momentum conservation
- HERWIG also has NLL α_S
- NLL expressions > 1

$e^+e^- \to Z \to$ **jets using** HERWIG-CKKW

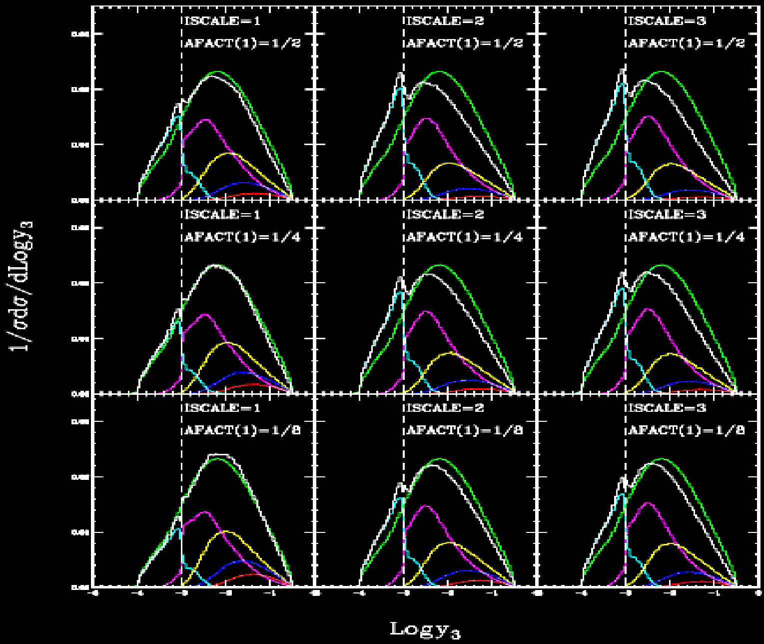
$Y_3 : \mathtt{Q}_0^2 = 2.88^2\ \mathrm{GeV}^2$



HERWIG 2 jets 3 jets 4 jets 5 jets 6 jets

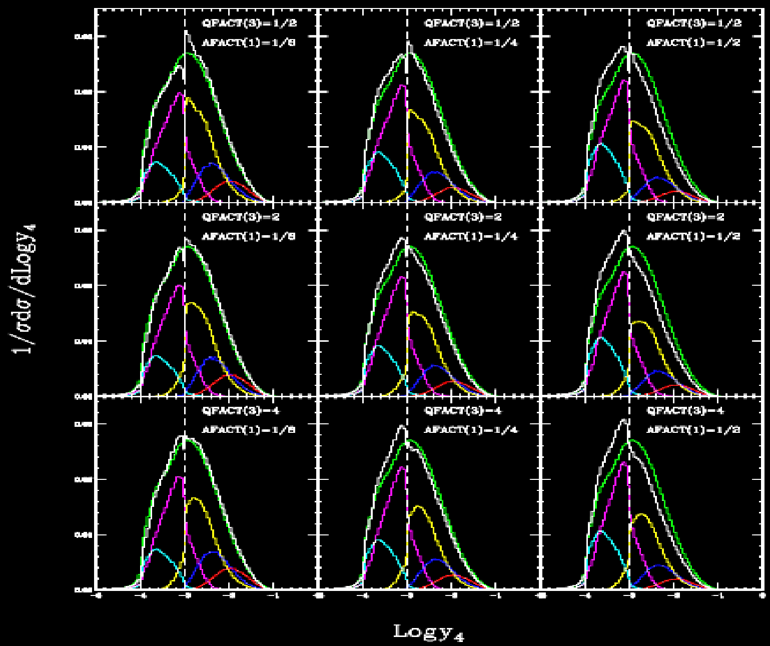
Varying prefactors for scale in Sudakov form factors and α_S

$k_T^2 \quad p_i \cdot p_j \quad (p_i + p_j)^2$



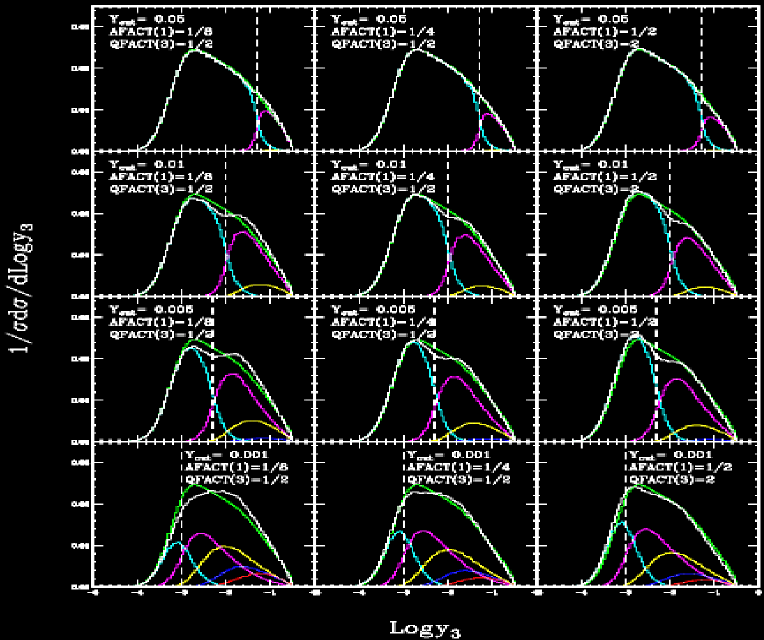
HERWIG 2 jets 3 jets 4 jets 5 jets 6 jets

Minimum scale



HERWIG 2 jets 3 jets 4 jets 5 jets 6 jets

HERWIG-CKKW (Hadron Level)



HERWIG 2 jets 3 jets 4 jets 5 jets 6 jets

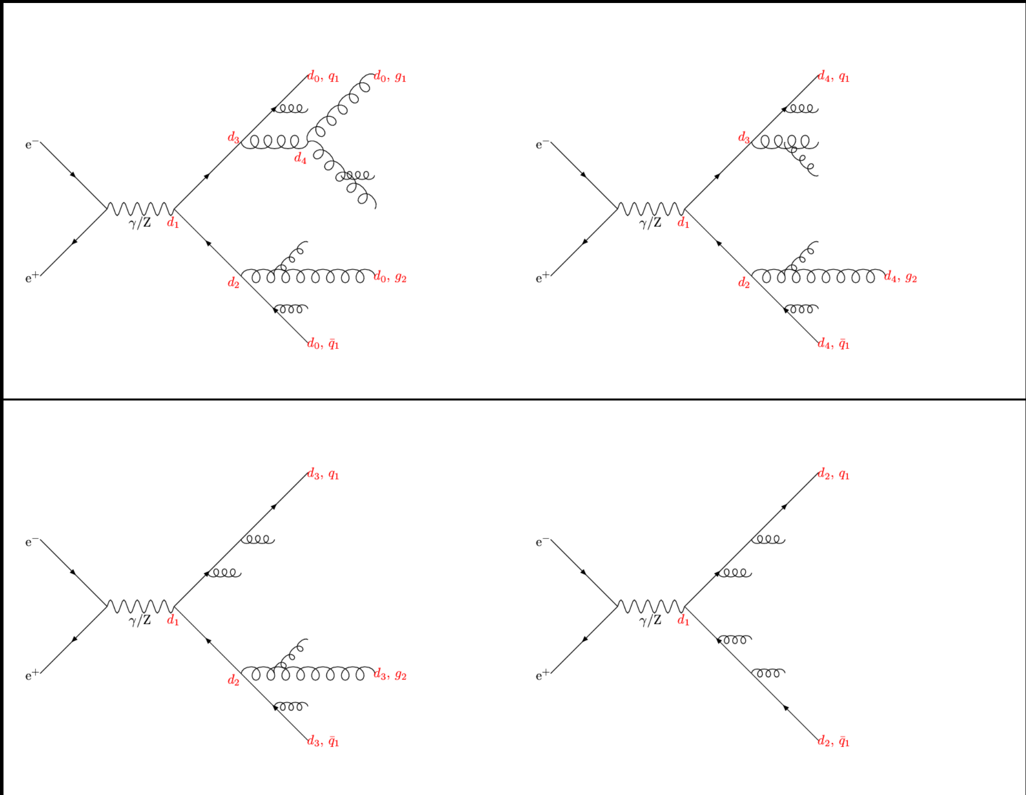
HERWIG-**CKKW Summary**

- HERWIG shower is not a NLL k_T shower
  - Sudakov weights not matched
- k_T values not preserved by shower
  - Events migrate above/below cutoffs
- Need different factors for hadron observables
  - Hadronization model should be married to shower

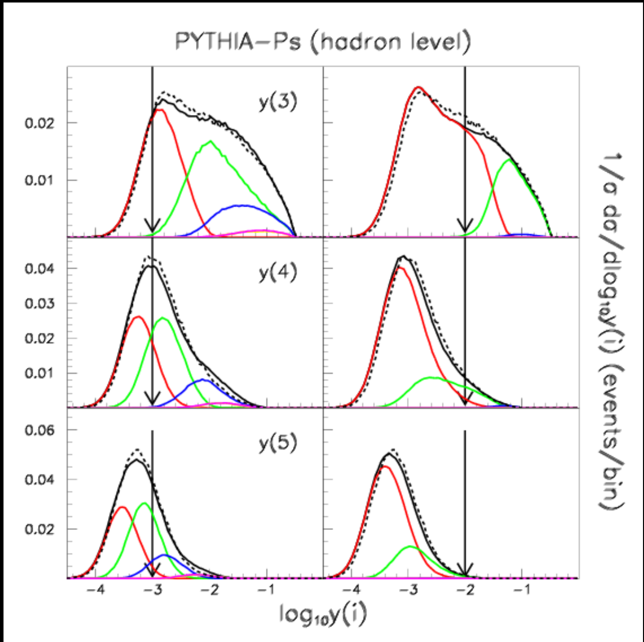
Pseudo–Shower Procedure

- Clustering
  - $d_{ij} = 2\left(\frac{E_i E_j}{E_i + E_j}\right)^2 (1 - \cos\theta_{ij}) = z(1-z)m^2$
- Sudakov form factor
  1. Cluster k partons using p_T scheme to get $\tilde{d}_i$
  2. PS k partons, vetoing emissions with $d > \tilde{d}_k$.
  3. Cluster again and throw away if $d_{k+1} > \tilde{d}_k$
  4. Use PS history to replace the 2 partons at scale $\tilde{d}_k$ with mother
  5. Continue until rejected or no partons left
- Choice of Scales
  - PYTHIA $= Q^2$, HERWIG $= p_i \cdot p_j$
- Treatment of Highest Multiplicity Matrix Element
  - In 1^{st} PS, allow radiation *as hard as* smallest $d_i > d_0$

Pseudo–Showers and Sudakov Weight

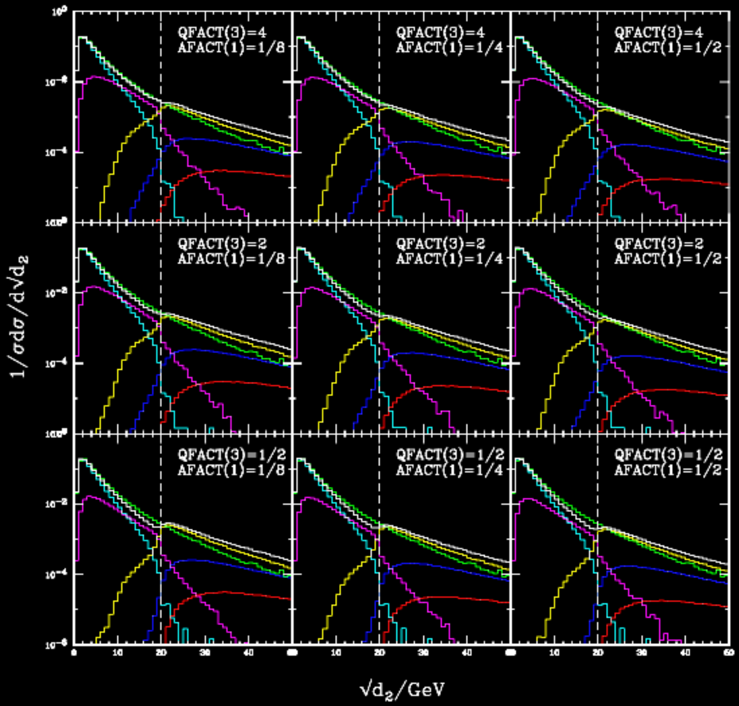


$e^+e^- \to Z \to$ jets using Ps-Sh



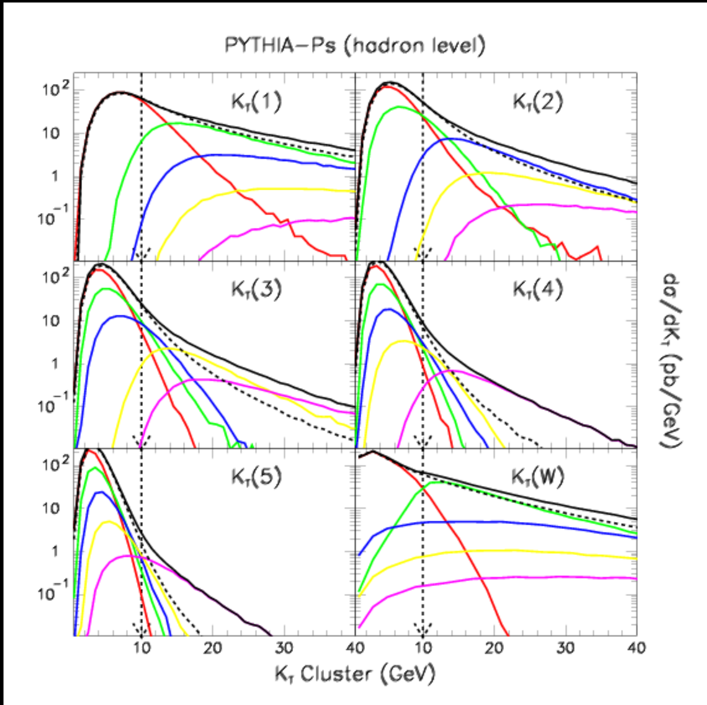
The matching scales: $10^{-3} \sim (2.88)^2$ GeV^2 and $10^{-2} \sim (9.12)^2$ GeV^2

W^+ (Tevatron) HERWIG-**CKKW** (Parton Level)

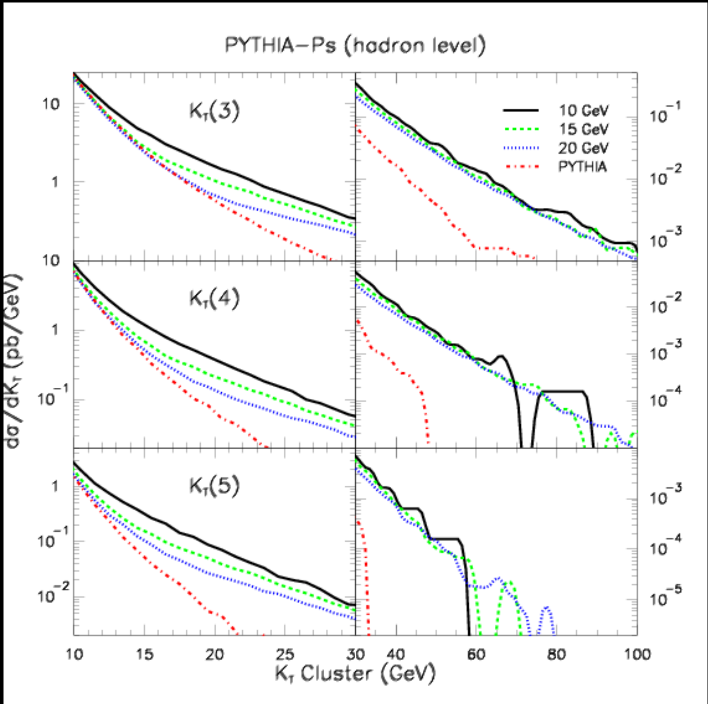


HERWIG 2 jets 3 jets 4 jets 5 jets 6 jets

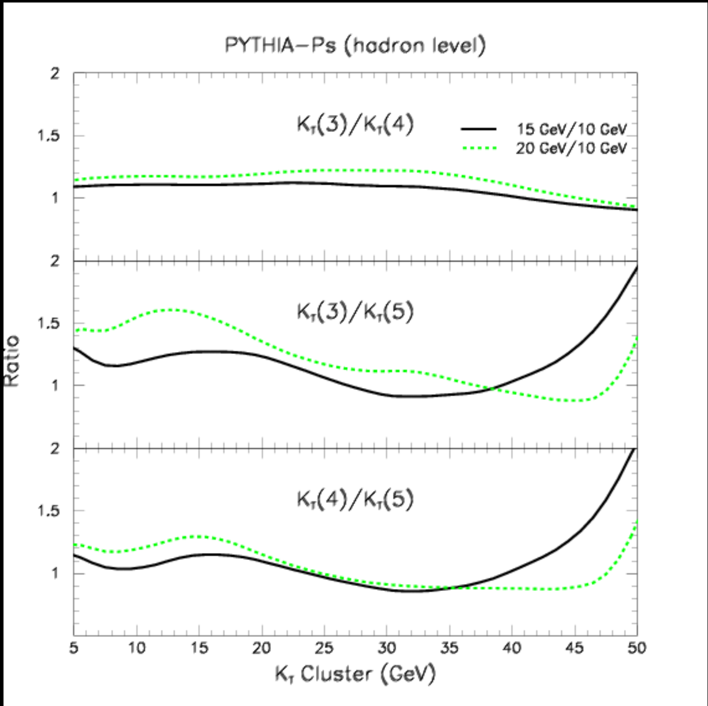
W^+ (Tevatron)


PYTHIA−Ps (hadron level)
K_T(1)
K_T(2)
K_T(3)
K_T(4)
K_T(5)
K_T(W)
10^2
10
1
10^-1
10
20
30
40
K_T Cluster (GeV)
dσ/dK_T (pb/GeV)

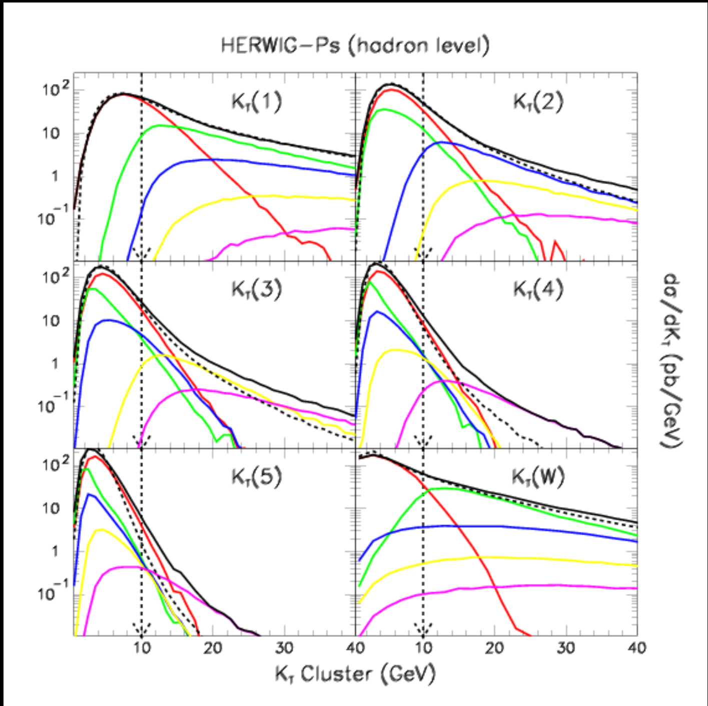
Variation with Cutoff



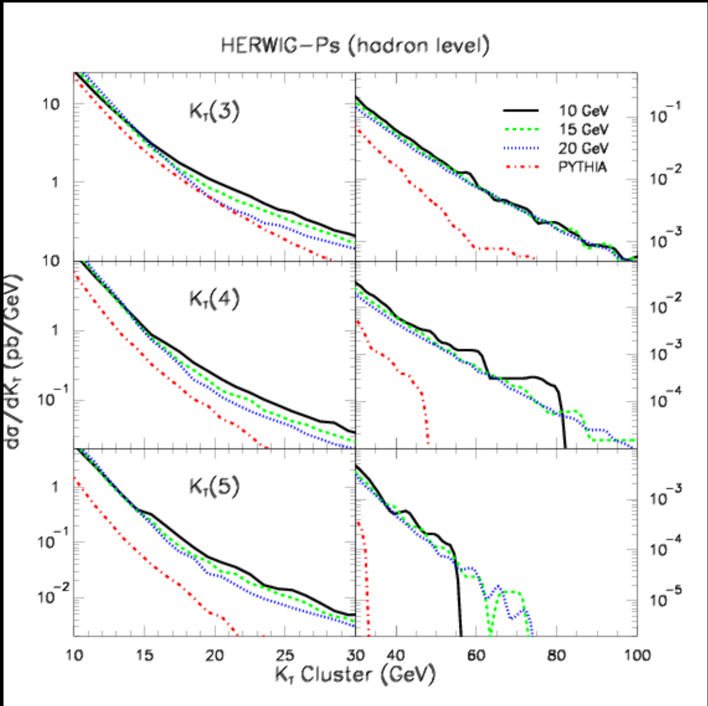
Ratios of Distributions



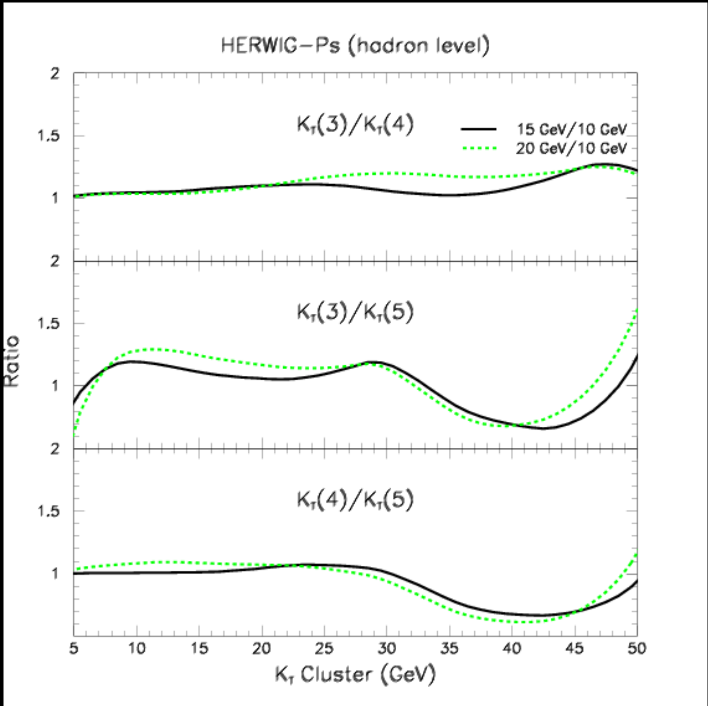
W^+ (Tevatron) (HERWIG)



Variation with Cutoff (HERWIG)



Ratios of Distributions (HERWIG)



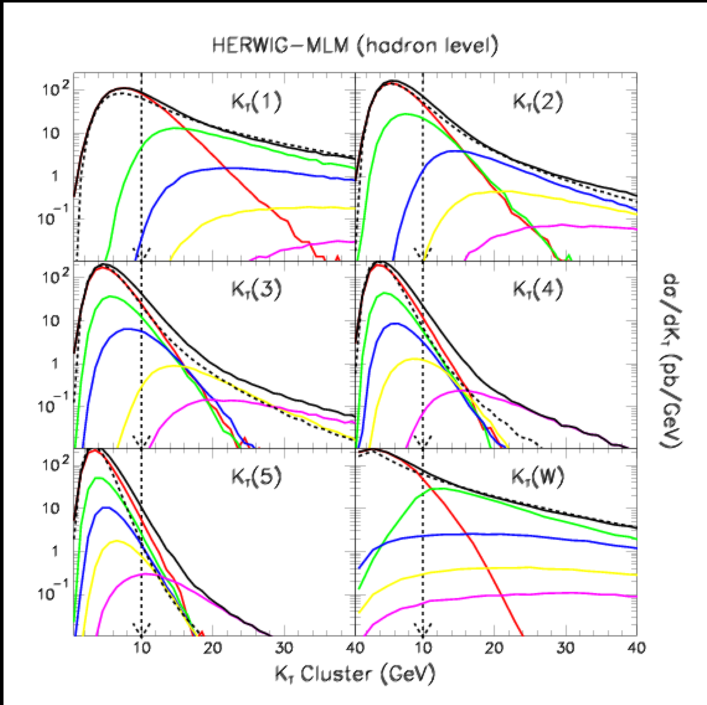
MLM method

1. Generate $W + n$ parton events of uniform weight
   - cuts on $|\eta^i| < \eta^{\max}$, $E_T^i > E_T^{\min}$, and $\Delta R_{ij} > R^{\min}$
2. Apply a PS using HERWIG
   - default scale is $\sqrt{p_i \cdot p_j}$, where i and j are color–connected partons.
3. Showered partons are clustered into N jets using a cone algorithm with parameters $E_T^{\min}$ and $R^{\min}$.
4. If $N < n$, the event is reweighted by 0. If $N \geq n$ (this is the *inclusive* approach), the event is reweighted by 1 if each of the original n partons is *uniquely* contained within a reconstructed jet. Otherwise, the event is reweighted by 0.

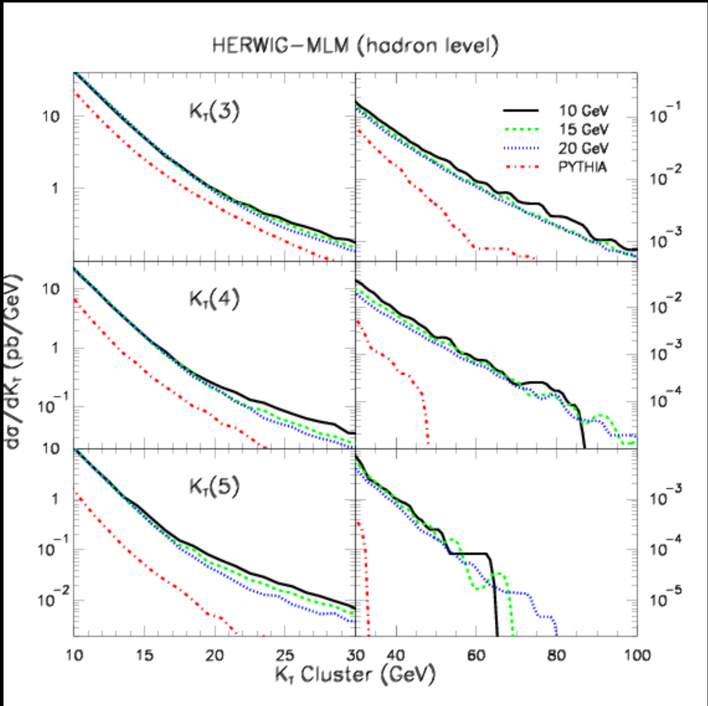
Comments

- Well motivated. It aims to prevent a PS from generating a gluon emission that is harder than any emission already contained in the "hard" matrix-element calculation.
- The cuts on E_T and ΔR play the role of the clustering cuts on k_T or p_T
- Rejection of events is like 1^{st} pseudo-shower
  - No internal Sudakovs
  - $\Delta(Q_h, Q_l)\ \alpha_s(q_T) \sim 1$?
- To make a direct comparison, replace cone variables with k_T
- Rejection replaced by $k_T^{n+1} < \tilde{k}_T^n$
- Add together different N's

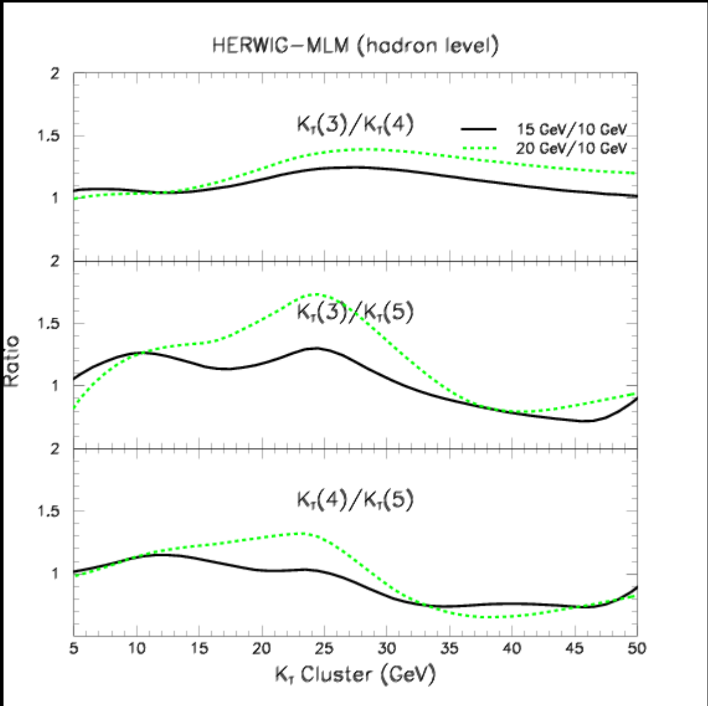
W^+ (Tevatron) (MLM-HERWIG)



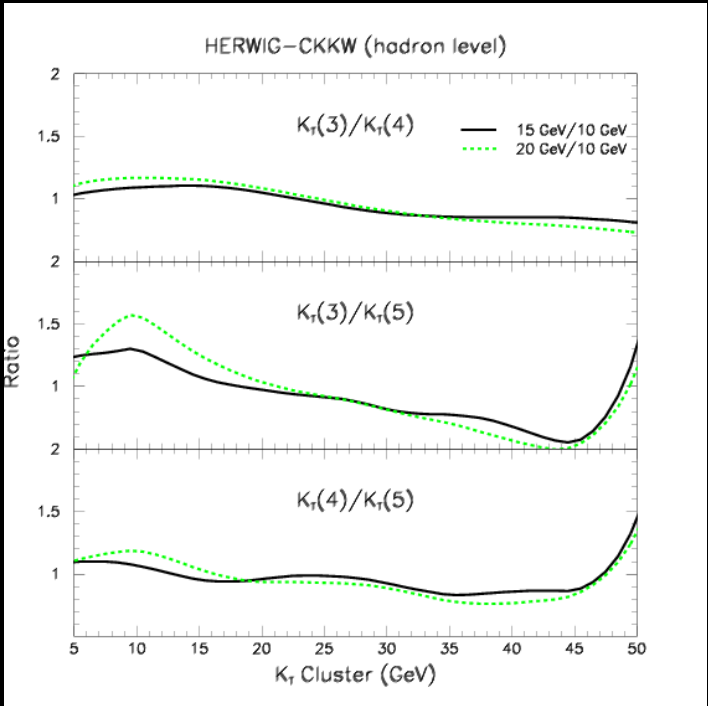
Variation with Cutoff (MLM-HERWIG)



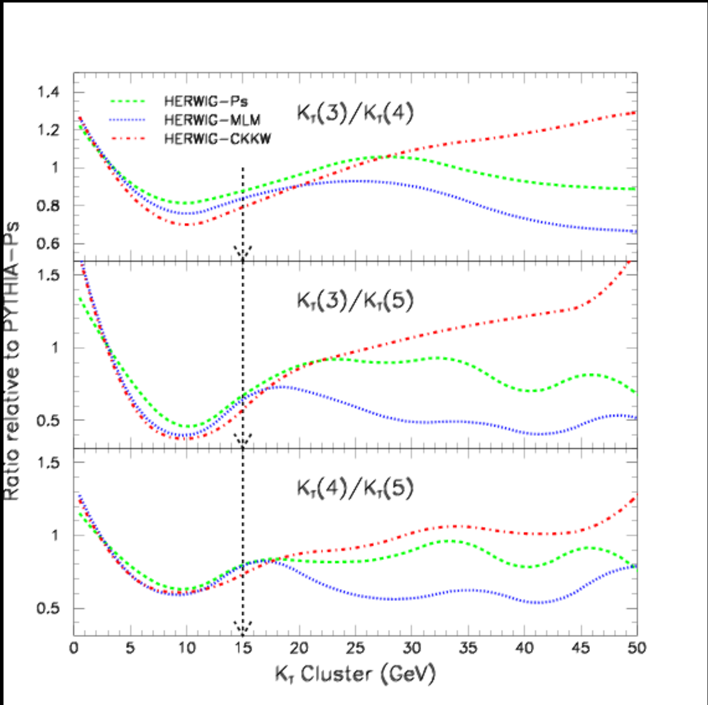
Ratios of Distributions (MLM-HERWIG)



Ratios of Distributions (CKKW-HERWIG)



Compare Ratios



matching scale of 15 GeV

Conclusions

1. Matching procedures studied here are robust to variation of the cut-off scale.
2. Relative distributions in k_T, for example, are reliably predicted.
3. The variation in the relative distributions from the three procedures depends on the variable. For variables within the range of the matrix elements calculated, the variation is 10-20%. For variables outside this range, which depend on the truncation of the matrix element calculation, the variation is larger 30-40%.
4. The subject of matching is far from exhausted. The procedures presented here yield an improvement over previous matching prescriptions, but they are *interpolations*.
5. Treatment of highest multiplicity needs work!

Direction

1. Fully hadronized W and Z events are available on `/pnfs/patriot`
2. W/Z + heavy flavor
3. Play with highest multiplicity
4. More theoretical developments

CDF Simulation